Remainder and Factor Theorem Proofs

When a polynomial is divided by (x-a) there is a quotient, Q(x) and remainder, r. i.e.

$$(x-a)\overline{)f(x)} \quad Q(x)+r$$

1. Put the steps A - E of each of the following proofs into the correct order.

2. Answer questions 1 and 2.

Remainder Theorem:

A polynomial f(x) has the remainder f(a) when it is divided by (x-a).

A $= r$

B i.e. $f(x) = (x-a)\,Q(x) + f(a)$

C $f(x) = (x-a)\,Q(x) + r$

D $= 0 \times Q(a) + r$

E $f(a) = (a-a)\,Q(a) + r$

Factor Theorem:

If f(a) = 0 and f(x) is a polynomial, then (x-a) is a factor of f(x).

A If $f(a) = 0$,

B ie (x-a) is a factor of f(x).

C $= (x-a)\,Q(x)$

D $f(x) = (x-a)\,Q(x) + f(a)$

E $f(x) = (x-a)\,Q(x) + 0$

1.

(i) Find the remainder when $3x^3 - 7x^2 + 8x - 3$ is divided by (x- 2)

(ii) Find k when $5x^2 + 3x - k$ gives a remainder of -1 when divided by (x+4).

2.

(i) Show that (2x +1) is a factor of $2x^3 + x^2 - 18x - 9$.

(ii) Which of the following are factors of $6x^3 - 43x^2 + 44x - 12$? Justify your answers

a) (x-6)

b) (x+1)

c) (3x-2)

Algebra level 8 ... use and prove the factor and remainder theorems. Maths Processes... prove or refute mathematical conjectures.

SSA Uncertainties

1. **Find the correct order for the steps A-H for each problem.**
2. **Decide which steps include (i) sources of errors.**
   (ii) propagation of errors.
   (iii) generation of errors.
3. **Justify your decisions in 2 (i - iii).**
4. **Explain why problem 2 has resulted in a value with such a large error.**

Problem 1

A The town council are letting a contract for construction of a new water tank for the town. Your firm has decided to put in a proposal for a water tank 5.3m high so that it will be hidden by existing trees. Calculate the diameter required in order for the tank to hold 500 $\pm$ 1 m^3 of water.

B $r = \sqrt{\frac{500}{5.3\pi}}$ errors: 0.20, 0.94, 1.14

C $r = 5.47989 \pm 0.03m$

D $V = 500 \pm 1 \ m^3$
$h = 5.3 \pm 0.05m$

E $d = 10.96 \pm 0.06\ m$

F $r = 5.47989 \pm 0.57\%$

G $r = \sqrt{\frac{V}{\pi h}}$

H $v = \pi r^2 h$

Problem 2

A A jet boat is heading back towards the jetty when the engine cuts out. Just before the engine stopped the boat was travelling at 15 m/s and was 36 metres from the jetty. If the water causes a deceleration of 3.1m/s², will the boat arrive at the jetty travelling at a safe speed? Use the formula;

$v^2_{final} = v^2_{initial} + 2ad$

a = acceleration
d = distance to travel

B $v_f^2 = 225 \pm 6.6\% + (-223.2 \pm 3\%)$

C $v_f = 1 \pm 8m/s$

D $v_f^2 = 15^2 + 2x(-3.1)x36$

errors: $v_i(\%)3.3$ a (%) 1.61
$v_i^2(\%)6.6$ +d (%) 1.39
2ad (%) 3.00

E $v_f^2 = v_i^2 + 2ad$
$v_i = 15 \pm 0.5m/s$
$d = 36 \pm 0.5m$
$a = -3.1 \pm 0.05m/s^2$

F $v_f^2 = 2 \pm 22$ (1100%)

G $v_f = 1.414 \pm 550\%$

H $v_f^2 = 225 \pm 15 + (-223.2 \pm 6.7)$

Measurement and Calculus Level 8 : ... compute the cumulative effect of uncertainties of measurement on quantities which are calculated using measured values.
..... evaluate uncertainties in a real context and describe their sources, generation and propagation.

Permutation and Combination Proofs

1. **Complete each proof by putting the numbered boxes in the correct sequence.**
2. **Try these proofs:** c) ${}^nC_r = {}^nC_{n-r}$ d) ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$ e) ${}^nC_r + 2\,{}^nC_{r-1} + {}^nC_{r-2} = {}^{n+2}C_r$
3. **Investigate: when does** ${}^nC_r = {}^nP_r$ **?**

A Prove ${}^{n+1}P_r = {}^nP_r + r\,{}^nP_{r-1}$	B Prove ${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$
1. $= n!(\frac{1}{(n-r)!} + \frac{r}{(n-r+1)!})$	1. $= \frac{n!(n+1)}{(r+1)!(n-r)!}$
2. $= \frac{n!}{(n-r)!} + \frac{rn!}{(n-(r-1))!}$	2. ${}^nC_r + {}^nC_{r+1}$
3. $= \frac{(n+1)!}{(n+1-r)!}$	3. $= {}^{n+1}C_{r+1}$
4. $= n!(\frac{(n-r+1)}{(n-r+1)!} + \frac{r}{(n-r+1)!})$	4. $= \frac{n!(r+1+n-r)}{(r+1)!(n-r)!}$
5. ${}^nP_r + r\,{}^nP_{r-1}$	5. $= \frac{(n+1)!}{(n-r)!(r+1)!}$
6. $= {}^{n+1}P_r$	6. $= \frac{n!}{r!(n-r)!} + \frac{n!}{(r+1)!(n-r-1)!}$
7. $= \frac{n!(n-r+1+r)}{(n-r+1)!}$	7. $= \frac{n!(r+1)}{(r+1)!(n-r)!} + \frac{n!(n-r)}{(r+1)!(n-r)!}$

Mathematical Processes:... prove or refute mathematical conjectures (Algebra level 8).

Sigma Proofs

1. The steps for two Sigma Proofs are muddled together below. Find the correct order for the two proofs.
2. Prove (i) $\sum(X+Y)=\sum X+\sum Y$
   (ii) $\sum(aX+bY)=a\sum X+b\sum Y$
   (iii) $\sum(X+Y)^2 \neq \sum X^2+\sum Y^2$
3. Discuss which of the Sigma Proofs is the most useful. Explain your reasoning.

A $=a(x_1+x_2+x_3+.......+x_n)$

B Prove $\sum aX = a\sum X$

C $=x_1+b+x_2+b+x_3+b+.......+x_n+b$

D $\sum aX =$

E Prove $\sum(X+b)=\sum X+nb$

F $=x_1+x_2+x_3+....+x_n+b+b+b+...+b$

G $=ax_1+ax_2+ax_3+....+ax_n$

H $=\sum xi+nb$

I $=a\sum X$

J $\sum(X+b)=$

K $=\sum X+nb$

Standard Deviation Proof

1. Complete the proof by finding the correct order for the boxes A-G.
2. Explain in words what is being done mathematically at each step of the proof.
3. Find out why another version of the standard deviation formula has n-l as the denominator rather than the n.

A $=\sqrt{\frac{\sum x_i^2-2\bar{x}\sum x_i+n\bar{x}^2}{n}}$

B $=\sqrt{\frac{\sum x_i^2-\sum 2x_i\bar{x}+\sum \bar{x}^2}{n}}$

C $=\sqrt{\frac{\sum x_i^2-n\bar{x}^2}{n}}$

D $=\sqrt{\frac{\sum x_i^2-2n\bar{x}^2+n\bar{x}^2}{n}}$

E $=\sqrt{\frac{\sum(x_i^2-2x_i\bar{x}+\bar{x}^2)}{n}}$

F $\sqrt{\frac{\sum(x_i-\bar{x})^2}{n}}$

G Prove $\sqrt{\frac{\sum(x_i-\bar{x})^2}{n}}=\sqrt{\frac{\sum x_i^2-n\bar{x}^2}{n}}$

Mathematical Processes: ..... improve or refute mathematical conjectures (level 8).

SSA Expectation Proofs

1. **Complete each proof by putting the numbered boxes into the correct sequence.**
2. **In words, describe what is happening at each step of the proof. eg. collecting like terms, expanding brackets etc.**
3. **Try these proofs: c)** $E(aX) = aE(X)$ **d)** $E(X+b) = E(X) + b$ **e)** $E(aX+b) = aE(X)+b$
4. **Investigation: Find or create a distribution that has:**
   (i) VAR (X) =E(X) **(ii)** VAR (X) < E(X) **(iii)** VAR(X) > E(X)

A Prove $VAR(X) = E(X^2) - \mu^2$

1. $= E\left[X^2 - 2\mu X + \mu^2\right]$
2. $= E(X^2) - \mu^2$
3. $= E\left[(X - \mu^2)\right]$
4. $= E(X^2) - 2\mu E(X) + E(\mu^2)$
5. VAR(X)
6. $= E(X^2) - 2\mu^2 + \mu^2$
7. $= E(X^2) - E(2\mu X) + E(\mu^2)$
8. $= E(X^2) - 2\mu\mu + \mu^2$

B Prove $VAR(aX+b) = a^2 VAR(X)$

1. $= E(a^2X^2) + E(2abX) + E(b^2) - a^2(E(X))^2$
   $-2abE(X) - b^2$
2. VAR (aX+b)
3. $= a^2\left[E(X^2) - \mu^2\right]$
4. $= E(a^2X^2 + 2abX + b^2) - \left[aE(X) + b\right]^2$
5. $= a^2E(X^2) + 2abE(X) + b^2 - a^2\mu^2 - 2ab\mu - b^2$
6. $= E\left[(aX+b)^2\right] - \left[E(aX+b)\right]^2$
7. $= a^2$ VAR(X)
8. $= a^2E(X^2) + 2ab\mu + b^2 - a^2\mu^2 - 2ab\mu - b^2$
9. $= a^2E(X^2) - a^2\mu^2$

Mathematical Processes:... prove or refute mathematical conjectures (Statistics level 8).

DSA

Simultaneous Equations
(3 unknowns)

1. Find the correct order for the steps A - I.

2. Find the correct order for the steps of each of the problems.

3. Make up your own real life problem which requires solving three equations. Swap with another group.

A Solving simultaneous equations with three unknowns. Read the question.

B Check your answer by substituting the three values into another of the original equations.

C You need to create two equations each with the same two variables. These will be numbered 4 & 5. Use the elimination method with two of the original equations.

D Eliminate the same variable again using a different pair of the original equations. If possible, equations 4 & 5 should be simplified.

E Write your answer as a sentence.

F Substitute the known value into 4. Calculate another value using this equation.

G Define the variables. Write three equations to represent the information. Number these equations to help show your working.

H Substitute the two known values into one of the original equations. Rearrange this to find the third value.

I Now you have two equations (4 & 5). Use elimination to find one of the values.

PROBLEM 1

1. In a large sports competition, points are scored for losses, wins and draws. The results for three teams are shown in the table.

	Team A	Team B	Team C
Losses	1	3	2
Draws	2	1	3
Wins	3	2	1
Total points	14	11	11

How many points are awarded for each type of result?

2. Eliminating y:

5 (4) $5y + 25z = 85$

(5) $-(5y + 7z = 31)$

$18z = 54$

$z = 3$

3. Let ; x = the number of points for a loss
y = the number of points for a draw
z = the number of points for a win

$x + 2y + 3z = 14$ (1) $3x + y + 2z = 11$ (2)

$2x + 3y + z = 11$ (3)

4. Eliminating x:

2 (1) $2x + 4y + 6z = 28$

− (3) $-(2x + 3y + z = 11)$

(4) $y + 5z = 17$

5. One point is awarded for a loss, two points for a draw and three points for a win.

6. x=1, y=2, z=3;
(2) 3+2+6= 1 1

7. y=2, z=3;
(1) x+4+9=14
x= 1

8. z=3;
(4) y+15=17
y=2

9. Eliminating x:

3 (1) $3x + 6y + 9z = 42$

− (2) $-(3x + y + 2z = 11)$

(5) $5y + 7z = 31$

PROBLEM 2

1. An amazingly intelligent physicist finds that three electrical currents are related by these equations.

$$3I_1 - I_2 - 5I_3 = 6$$
$$-I_1 + 5I_2 - 3I_3 = 12$$
$$-I_1 - 9I_2 + 25I_3 = -16$$

How big is each of the currents, $I_1 I_2$ and I_3?

2. Eliminating x:

❶	$3x - y - 5z = 6$
+3 ❷	$+ (-3x + 15y - 9z = 36)$
❹	$14y - 14z = 42$
	$y - z = 3$

3. Eliminating y:

2 ❹	$2y - 2z = 6$
+ ❺	$+ (-2y + 5z = -3)$
	$3z = 3$
	$z = 1$

4. z=1;
❹ y −1=3,
y=4

5. y=4, z=1;
❶ 3x −4 −5=6
x=5

6. The three currents are 5 amps, 4 amps and 1 amp respectively.

7. Eliminating x:

❶	$3x - y - 5z = 6$
+ 3❸	$+ (-3x - 27y + 75z = -48)$
❺	$-28y + 70z = -42$
	$-2y + 5z = -3$

8. x=5, y=4, z=1;
❷ −5+20-3=12

9. Let x=the number of amps for I_1
y=the number of amps for I_2
z=the number of amps for I_3
❶ 3x − y − 5z=6
❷ − x+5y − 3z=12
❸ − x − 9y + 25z= −16

Time Savers

1. **Sequence the instructions and one problem only. Different groups sequence different problems.**
2. **Give the correct instruction sequence. Groups then order the problems to match the instructions.**

PROBLEM 3

1. Tom, Dick and Harry decide to go to a music sale. They each bought some CDs, cassettes and records. Their maths teacher asked what the sale prices were and they gave him this information.

Tom spent $58.50 and had bought one CD, two cassettes and one record. Dick spent $75 and had bought two CD'S, one cassette and two records. Harry spent $85 and has one, three and two of each respectively!

What were the prices of each item?

2. y=14;
❹ − x+28=10
x=18

3. x=18,y=14;
❶ 18+28+z=58.50
z=12.50

4. Eliminating x:

❹	$-x + 2y = 10$
+ ❺	$+ (x + y = 32)$
	$3y = 42$
	$y = 14$

5. x=18, y=14, z=12.50
❷ 36+14+25=75

6. Eliminating z:

❸	$x + 3y + 2z = 85$
− ❷	$- (2x + y + 2z = 75)$
❹	$-x + 2y = 10$

7. Let; x=the price of a CD
y=the price of a cassette
z=the price of a record
❶ x+2y+ z= $58.50 ❷ 2x+y+2z=$75
❸ x+3y+2z=$85

8. Eliminating z:

2 ❶	$2x + 4y + 2z = 117$
− ❸	$- (x + 3y + 2z = 85)$
❺	$x + y = 32$

9. CD's cost eighteen dollars, cassettes cost fourteen dollars, records cost twelve dollars and fifty cents.

Algebra level 8:...use simultaneous equations to model real and simulated situations, and interpret their solutions in a given context.

DSA Linear Programming

1. **Find the correct order for the steps A-J.**
2. **Find the correct order for the steps of each of the problems. Discuss other factors within the practical situations which may affect the optimal solution. (eg. some staff working more slowly than others).**
3. **Make up your own real life problem which can be solved using linear programming. Swap with another group.**

A Define the variables. To do this look for the question word such as how many, find....... Describe the variables precisely.

B Use the equation that is to be minimised. Replace the variables with the numbers from the vertices. Do this for each vertex separately.

C Write down key information

D Write the conditions as inequalities. Check that these are in the simplest form. Cancel if possible. Check for conditions that are not stated.

E Read all of the question.

F Graph the conditions.

G Look for the information that limits the situation. The conditions may include limits on time, space, cost, etc.

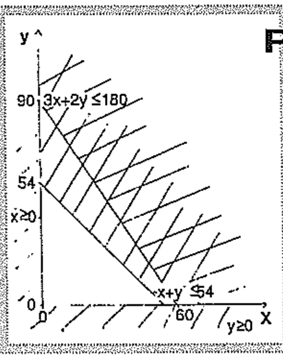
H Decide which is the best answer. Sometimes the maximum or minimum value is asked for as well. Write the best solution in a sentence.

I Write the equation to be maximised or minimised. This equation can start with a word eg. Profit = ,Cost=...

J List the vertices. Sometimes whole number values give a more sensible answer.

PROBLEM 1

1.



2. A hairdryer manufacturing firm makes two models, the 'Excel' and the 'Drifast'. The firm employs 5 trained and 6 untrained employees who work a three hour half-day. The Excel model takes 15 minutes of trained work and 20 minutes of untrained labour. The Drifast model needs 10 and 20 minutes respectively. How many of each should be produced every half day for maximum profit if the firm makes $15 on each Excel and $10 on each drifast?

3. **Condition on workers time;**
trained worker minutes=900
untrained worker minutes=1080

4. In order to maximise their profit the firm should produce 54 Excel hairdryers and no Drifast hair dryers, in each half day of business.

5. Excel and Drifast models
5 skilled employees
6 unskilled employees
3 hour half day.
Excel: 15 minutes trained
20 minutes untrained
Drifast: 10 minutes trained
20 minutes untrained.

6. (0,0), (0,54), (54,0)

7. $\left.\begin{array}{l}15x+10y \le 900\\ 20x+20y \le 1080\end{array}\right\}$ $\begin{array}{l}3x+2y \le 180\\ x+y \le 54\\ x \ge 0\\ y \ge 0\end{array}$

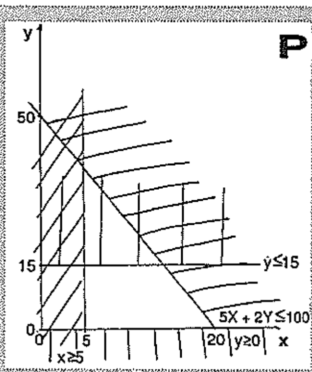
8. Profit = $15x+10y$

9.

Vertex	Profit
(0,54)	$540
(54,0)	$810

10. x = the number of Excel models made
y = the number of Drifast models made

PROBLEM 2

1. 

2. x=the number of couches bought
y=the number of chairs bought

3. Cost= \$650x + \$250y

4. James has won a contract to furnish a corporate lounge. He has to fill 100m² of seating space with a combination of couches and chairs. The business has set requirements that there must be at least five couches and no more than fifteen chairs. Couches take up 5m² and cost \$650 while chairs require 2m² of seating space and cost \$250. How many couches and chairs should James buy in order to minimise the cost while satisfying the requirements of the corporation?

5. The cheapest solution is to use 5 couches and no chairs. The next best solution costs \$3,750 more and would include 5 couches and 15 chairs.

6. Conditions from the company: at least five couches no more than 15 chairs
condition on space (100m²)

7. **Couches:** 5m² space, \$650 — **Chairs:** 2m² space, \$250
total space is 100m²

8.

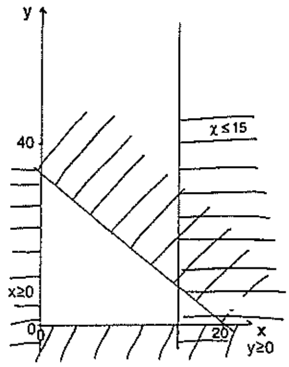
Vertex	Cost
(5,15)	\$7000
(14,15)	\$12,800
(20,0)	\$13,000
(5,0)	\$3250

9. (5,0)
(5,15)
(14,15)
(20,0)

10. $x \geq 5$
$y \leq 15$
$5x + 2y \leq 100$
$y \geq 0$

PROBLEM 3

1. A boarding kennel looks after large and small dogs. They have a policy of keeping no more than 15 large dogs at a time. The profit for each large dog is \$4 a day and for small dogs is \$2. Feeding the small dogs costs 30 cents each and the large dogs cost 50 cents. Food bills must not exceed \$10 - what is the maximum profit achieveable and how can it be done?

2. 

3. Profit = 4x+2y

4. **Large dogs:** profit \$4, 50c to feed, no more than 15
Small dogs: profit\$3, 30c to feed
\$10 available for food

5. $50x + 30y \leq 1000$
$5x + 3y \leq 100$
$0 \leq x \leq 15$
$y \geq 0$

6. (0,0)
(0,33)
(15,0)
(15,8)

7. **Condition on cost:**
no more than \$10
Conditions on the number of dogs:
no more than 15 large dogs

8. x=the number of large dogs kept
y=the number of small dogs kept

9. The maximum achieveable profit for the kennels is \$76. This is done by boarding fifteen large dogs and eight small dogs.

10.

Vertex	Profit
(0,33)	\$66
(15,8)	\$76

Standard Normal Distribution

1. **Find the correct order for the steps A-G.**
2. **Find the correct order for the steps of each of the problems. Each group must discuss part (ii) of each question and decide on their own answer and justification.**
3. **Make up your own real life problem which requires using a standard normal distribution. Swap with another group.**

A Write the parameters of the distribution. For a normal distribution the parameters are the mean and the standard deviation.

B Find what the questionis asking for. This is usually near the end. Write it as a mathematical statement.

C Look up the z values in the tables. Use the symmetry of the curve to work out the probability. You may have to add or subtract.

D Read the whole question.

E Sketch the distribution on a bell curve. Shade the region that is required to solve the problem.

F Change the parameters and values to those of the standard normal distribution. This has a mean = 0 and standard deviation =1.

G Write the answer as a sentence.

Problem 1

1. Sylvia is delighted that the wait time for clients at her hair salon has only a mean of 2.5 minutes and a standard deviation of 27 seconds. Past experience has shown the wait time is normally distributed. (i) What proportion of clients will have to wait more than three minutes?
(ii) Design a sign that would explain the expected wait times to clients.

2. The proportion of clients that have to wait more than three minutes =?
To find this out we need $p(X > 3) = ?$

3. Let X = wait tine (in minutes)
$\mu = 2.5$ minutes
$\sigma = 27$ seconds
$= 0.45$ minute

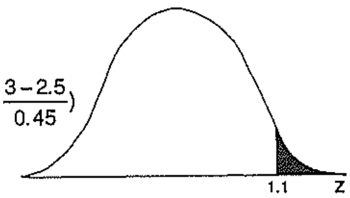
4. $p(Z > 1.1)$
$= 0.5 - 0.3665$
$= 0.1335$

5. 13.35% of clients at the salon will have to wait more than three minutes before they have their hair cut.

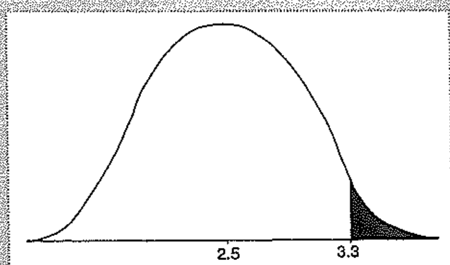
6. $z = \frac{x - \mu}{\sigma}$

$p(X > 3) = p(Z > \frac{3 - 2.5}{0.45})$

$p(Z > 1.1)$



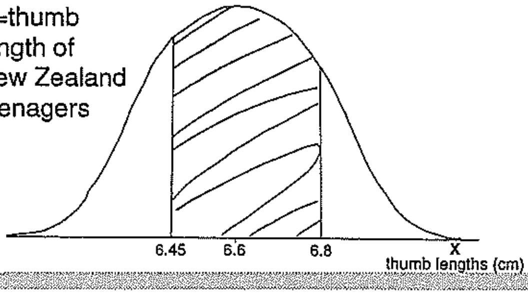
7. X- wait times for clients



wait times (minutes)

Problem 2

1. X=thumb length of New Zealand teenagers

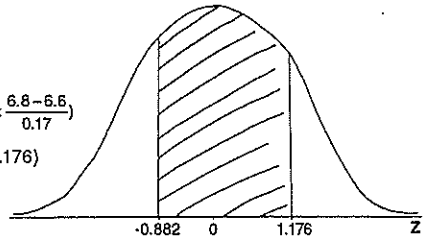


2. $Z = \frac{X-\mu}{\sigma}$

$p(6.45 < X < 6.8)$

$= p(\frac{6.45-6.6}{0.17} < Z < \frac{6.8-6.6}{0.17})$

$= p(-0.882 < Z < 1.176)$



3. The expected number of teenagers with thumbs between 6.45 and 6.8cm = ?
To find this out we need p(6.45<X>6.8)=?

4. Let X = thumb length of NZ teenagers
μ = 6.6 cm
σ = 0.17cm

5. 0.6916 of the group would be expected to have thumbs between 6.45 and 6.8cm long.
This is 83 people.

6. $p(-0.882 < Z < 1.176)$
= 0.3106 + 0.3810
= 0.6916

7. Thumb lengths for New Zealand teenagers are normally distributed with a mean length of 6.6 cm and standard deviation of 0,17cm.

(i) In a group of 120 teenagers, how many would be expected to have thumbs between 6.45 and 6.8 cm long?

(ii) Is the result realistic?

Time Savers

1. Sequence the instructions and one problem only. Different groups sequence different problems and explain their answers to the class.
2. Give the correct instruction sequence. Groups then order the problems to match the instructions.

Problem 3

1. $p(x < 130) = ?$

2. The probability that the florist uses less than 130 rolls in a year is 0.0087.

3. A florist knows that the number of rolls of ribbon he uses follows a normal distribution.
On average he uses twelve rolls a month with a standard deviation of 1.7 rolls.

(i) Find the probability that the florist uses fewer than 130 rolls in one year.

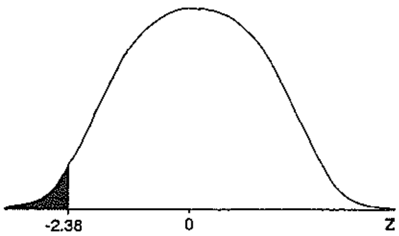
(ii) Make, and justify a prediction on how many rolls he will have to buy in one year.

4. $Z = \frac{X-\mu}{\sigma}$

$p(X < 130)$

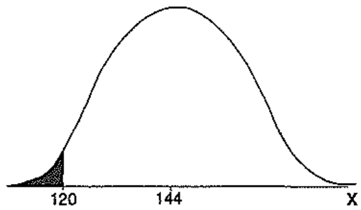
$= p(Z < \frac{130-14}{5.89})$

$= p(Z < -2.38)$



5. Let X = the number of rolls of ribbon used each year.
μ = 144 rolls
σ = 5.89 rolls

6. X = the number o
of ribbon used in



7. $p(z < -2.38)$
$= 0.5 - 0.4913$
$= 0.0087$

(sums of normal distributions)

Statistics Level 8: ... solve problems using techniques which stimulate a probability situation.... choose the appropriate distribution to model a given situation, calculate probabilities, and expected values, and make predictions using the model.

Confidence Intervals

1. **Find the correct order for the steps A - G.**
2. **Find the correct order for the steps of each of the three problems. Decide whether each is an appropriate use of confidence intervals and whether each of them provides a realistic model for the situation it describes.**
3. **Make up your own real life problem which requires using confidence intervals.**
4. **Plan an investigation that would allow you to estimate accurately the margins of error for your real situation in 3. Comment on (i) how to choose a sample (ii) what information to collect (iii) how to use the data obtained.**

A Use the normal distribution tables to find k, the confidence coefficient. (Sometimes k is written as $Z^{-1}_{\alpha/2}$)

B Write the answer as a sentence.

C Read the whole question

D Substitute the values into the formula. Use the sample values if the population values are unknown.

E Find what the question is asking for. This is often near the end of the question.

F Write down the key information from the question.

G Write out the formula for the confidence intervals.

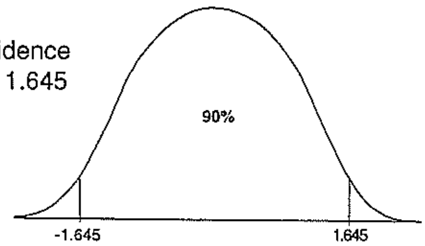
Problem 1

1. $n = 50$ babies
$\bar{x} = 3.61$ kg
$s = 0.39$ kg

2. We are 90% confident that the mean weight of Wellington babies lies between the values 3.52 kg and 3.70kg.

3. A sample of 50 babies was used to estimate the mean weight of babies born in Wellington. The sample statistics were found to be $\bar{x} = 3.61$ kg with a standard deviation of 0.39 kg. Find the 90% confidence intervals for the true mean weight of Wellington babies.

4. for 90% confidence intervals, $k = 1.645$



5. We need the 90% confidence intervals for μ, the mean weight of Wellington babies.

6. $\bar{x} - k\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + k\frac{\sigma}{\sqrt{n}}$

7. $3.61 - 1.645\frac{0.39}{\sqrt{n}} < \mu < 3.61 + 1.645\frac{0.39}{\sqrt{50}}$

$3.61 - 0.09 < \mu < 3.61 + 0.09$

$3.52 < \mu < 3.70$

(mean)

Problem 2

1. $p - k\sqrt{\frac{p(1-p)}{n}} < \pi < p + k\sqrt{\frac{p(1-p)}{n}}$

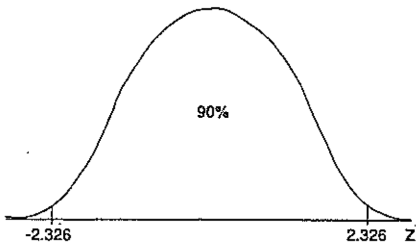
2.
$0.5832 - 2.326\sqrt{\frac{0.5832 \times 0.4168}{1250}} < \pi < 0.5832 + 2.326\sqrt{\frac{0.5832 \times 0.4168}{1250}}$

$0.5832 - 0.0324 < \pi < 0.5832 + 0.0324$

$0.5508 < \pi < 0.6156$

3. We are 98% confident that the proportion of all Auckland school leavers that get jobs before April the next year lies within the range 55 to $61\frac{1}{2}$%.

4. For 98%, confidence intervals, k = 2.326



5. We must find the 98% confidence intervals for π the proportion of Auckland school leavers to get jobs before April the following year.

6. A sample of 1250 Auckland school leavers showed that 729 of them found a job before April of the following year. Find 98% confidence intervals for the proportion of all Auckland school leavers to find jobs before April.

7. $n = 1250$ people

$p = \frac{729}{1250} (= 0.5832)$

(proportion)

Statistics Level 8: ... evaluate and explain the meaning of confidence intervals in estimating population parameters and in using samples for quality control.

Time Savers

1. Sequence the instructions of one problem only. Be prepared to justify your answer to the class.
2. Give the correct instruction sequence. Groups then order the problems only.
3. Choose one numbered box from a problem. Complete the answer using that step as a starting point.

Problem 3

1. Stephan wants to know which concert to go to in order to get the best value for money.

2. Stephan is a music fan, but he also likes getting value for money. He is deciding which of two concerts to attend. Use the 95% confidence intervals for the difference of the mean lengths of two similar concerts to decide which he should attend. Stephan has collected data from the last ten concerts of each band.

BAND	$\bar{x}$	s
SUPER 3	92 mins	4 mins
MODULAA	87 mins	7.3 mins

3.
$92 - 87 - 1.96\sqrt{\frac{16}{10} + \frac{53.29}{10}} < \mu_D < 92 - 87 + 1.96\sqrt{\frac{16}{10} + \frac{53.29}{10}}$

$5 - 5.159 < \mu_D < 5 + 5.159$

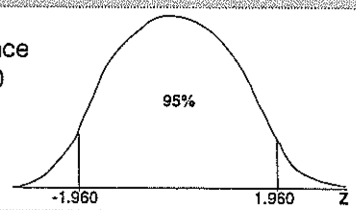
$-0.159 < \mu_D < 10.159$

4. Stephan would probably get more value for money at the Super 3 concert. However zero is included within the 95% confidence intervals so there may be no difference in the mean lengths of the concerts.

5. $\bar{x}_1 - \bar{x}_2 - k\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_D < \bar{x}_1 - \bar{x}_2 - +k\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

(μ_D = difference between the means)

6. for 95%, confidence intervals, k=1.960



7.

SUPER 3	MODULAA
$n_1 = 10$	$n_2 = 10$
$\bar{x}_1 = 92$ minutes	$\bar{x}_2 = 87$ minutes
$s_1 = 4$ minutes	$s_2 = 7.3$ minutes

(difference of means)

Task

You are to form a group application to submit to the statistical funding agency STATFUND.

You must choose one of the hypotheses on page 48 to use as the basis of your statistical experiment.

You are to design an experiment that will enable you to gather data which will help show whether the statement is true or false.

The letter below explains what you must do.

SF

STATFUND INDUSTRIES
732 Solwed Ave
Sindrew
Auckland
New Zealand

Thank you for the interest you have shown in our statistics grants. Explained below are the requirements you must supply in order to be considered for funding. Your application may be submitted in written, video or oral form.

Submissions for funding grants must include;

- *a clear explanation of the study to be undertaken. This should include comments regarding the general usefulness of the experiment.*
- *a clear explanation of how the data/information will be obtained. This should include sampling techniques; any scoping activity that will be undertaken, and the way in which the data will be collected (eg. by telephone, interview, questionnaire etc.), use of repeated data collection, etc*
- *specifically what data/information will be collected. (include copies of survey forms, enquiry letters etc.)*
- *discussion on the expected limits of accuracy of your results with explanations and comments on any likely sources of bias.*
- *a proposed timeline showing the stages of your experiment from the proposal submission through to completion of the project.*

Good luck with compiling your proposal. We look forward to its receipt.

Sally Murchison

Director of Research

Experiment Choices

1. There are more New Zealand made television programmes on air now than ever before.
2. People who are overweight suffer from high blood pressure.
3. Female students tend to out perform male students in internally assessed courses.
4. More cars use unleaded petrol than other fuels.
5. Use of walkmans can damage hearing.
6. Taking part in active and passive sport is New Zealanders' favourite pastime.
7. The crime rate in New Zealand is higher than it was ten years ago.
8. People who work in the city centre have more health problems than people who don't.
9. Magazine sales in New Zealand rose by more than 10% last year.
10. Roller blading is a cheaper sport to take up than skating or tennis.
11. Twenty five percent of adults read a newspaper each day.
12. Most textbooks have more boy-friendly questions than girl-friendly questions.
13. Skin cancer rates in New Zealand are different for different locations.
14. People of different ethnic groups have different shaped feet.
15. Drink drive campaigns have no effect on people's attitudes or on the road toll.
16. Third formers have more positive attitudes towards school than seventh formers.
17. There are more stay-at-home fathers caring for children now than there were five years ago.
18. More women than men work for charity organisations.
19. Chess is a game played by people who are above average at mathematics.
20. New Zealand music bands are selling more records overseas now than was the case ten years ago.
21. 10% of New Zealand cars have personalised number plates.
22. Own choice.

Statistics level 8:...plan a statistical investigation to estimate (2σ) confidence itervals (margins of error) for the estimation of population parameters.

Simulations

Task

1. **Read the steps for carrying out a probability simulation.**
2. **Use the instruction steps to help complete a simulation problem. Choose one of the simulation problems 1-9 to carry out. To save time different groups can work on the same simulation and share results.**
3. **Where possible decide which probability distribution best models the situation you have simulated. Compare simulation results with theoretical results.**
4. **Extend your simulation to obtain more general results.**

- clearly define the problem.
- decide on a suitable simulation method eg. spinner, cards, coin, random numbers, computer, die etc.
- decide on a clear, simple recording method.
- carry out a simulation of the situation.
- repeat the simulation a large number of times recording results as you go.
- summarise and analyse the results.
- comment on the limitations of the simulation model.
- clearly state the solution to the problem.

1. Ice Cream

The yummy ice cream company claims that half of the spoonfuls of their hokey pokey ice cream include one chunk of hokey pokey. How many spoonfuls do you need to have to be sure of getting four chunks?

Use a coin for the simulation. Each throw represents a spoonful of ice cream. (head=chunk, tail =no chunk)

2. The Big O.E.

Travellers in India have a chance of one in four of succumbing to a stomach illness while there. You are to travel there with four others. Find the likelihoods of different numbers of you getting sick on the trip. Use a paper clip as a spinner to simulate whether each person gets sick or stays well.

3. Men or Women

Two thirds of Primary school teachers are women. In a school of eight teachers, what is the expected number of female teachers?
Use a die for the simulation. Each throw represents one of the teachers. (1-4 means female, 5 or 6 means male). Throw eight times to find how many women in one simulated school. Repeat throwing to continue the simulation.

4. Weetbix Cards

There are ten different weetbix cards in a new set of cards about tropical fish. How many boxes of weetbix do you need to buy to be sure of collecting (i) half a set, (ii) a full set.

Use random numbers to simulate this situation. Each digit represents a different card.

5. Naughts and crosses

Design a simulation that will allow you to find the average number of moves for the game of noughts and crosses.

6. Birthdays

Use one suit without the ace from a pack of cards to simulate zodiac signs of a group of four people who work together in a cafe. How likely is it that two of them have the same astrological sign?

7. Offspring

Design a simulation to predict the number of offspring a pet cat will have in the next litter.

8. Radioactive Decay

Design a simulation to model radioactive decay. Each atom may have a 1/10 chance of having decayed in the period of time being measured.

9. Intersections

Design a simulation that will enable you to decide whether the wait time for cars is longer at a traffic light intersection or at a roundabout.

LMA Language Mathematics Activities

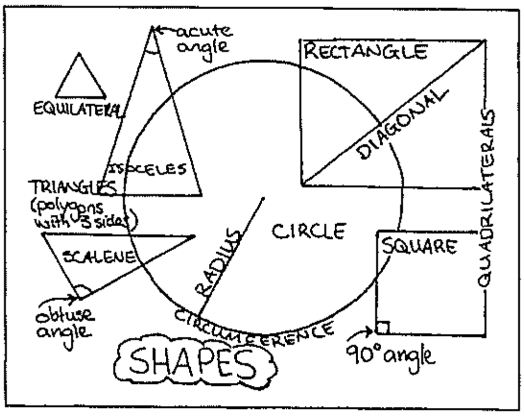
These activities focus on the language aspect of mathematics. They aid understanding of the vocabulary used in different topics.

The word lists can be used in many ways. There are possibilities for creative written and oral communication, and the words can be used in groupwork or by individuals. The activities also provide opportunities for self evaluation by the learner and for tutor assessments.

Some ways of using the activities are listed below:

1. Display all the words from a topic list on one large sheet of paper. Group the words according to similar uses or meanings, and set it all out in a logical manner. Use arrows, diagrams, graphs, explanations etc to help make the meanings of the words clear.

eg.



2. Play a whole class game in two or more teams. Points are scored for each accurate description given. The teacher judges how many points each given description is worth:

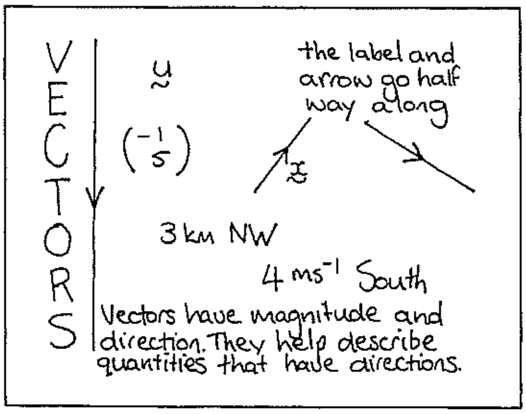
- 2 points for a full description.
- 1 point for a partial description.
- 0 points if no description is given.

Teams can take turns to choose which word they will define. Answers can be given by individuals, or alternatively, by the group working together.

3. Complete a written topic summary using the list as a starting point.

4. Make a poster to illustrate the meaning of one or more words from the list. Different students can complete one poster for each word as a class project.

eg.



5. Prepare a short talk to explain the meaning and uses of one of the words, or a small group of words from the list.

6. Put the words into two groups. The first group is the words that the students feel they have a thorough understanding of. The second group is the words that will require more explanation or revision.

7. A group of students divides evenly the words amongst themselves. Each student in the group has a set of words and the responsibility for ensuring all other students in their group fully understand each of their share of the word list.

LMA Other Language Mathematics Activities

➢ Research the history of the mathematical theory in a topic, for example probability theory, integration, set theory, trigonometry, statistical graphing, differential equations etc.

➢ Write an essay about a female or a male mathematician. Either choose a historical figure of someone currently working in a mathematical occupation. Write about the person and about the mathematics they use or used.

➢ Keep a mathematics journal. Express your feelings about mathematics, keep summaries of recent work learnt, interesting puzzles and investigations. Keep a record of mathematical articles from the newspaper.

➢ Write a discussion that might take place between the four mathematicians on the cover.

➢ Write a poem or a story about mathematics or with a mathematical theme.

➢ Write your own mathematical problems and swap with others to solve them.

➢ Write an essay about whether or not mathematics is a more important subject today than it has ever been. Include reasons that support your belief.

➢ Find out what is happening in mathematics and mathematics research locally and nationally.

Uncertainties

MODEL ERROR

MANTISSA

RELATIVE ERROR

PROPAGATION

ADJUSTMENT

ROUNDING

CALCULATION ERROR

PERCENTAGE ERROR

MEASUREMENT ERROR

NORMALISED FLOATING POINT

TRUNCATING

SOURCES OF ERRORS

BATCH ADDING

ACCURACY

ABSOLUTE ERROR

Algorithms

REPETITION

FLOW DIAGRAM

LOGIC DIAGRAM

INPUT

DECISION-MAKING

INITIALISE

STRUCTURE DIAGRAMS

STOP

BECOMES

ORDERED STEPS

ITERATION

ARROWS

OUTPUT

LOOP

DECISION BOX

INSTRUCTION

PROGRESS FROM LEFT TO RIGHT AND TOP TO BOTTOM

DESK CHECK

START

$X \leftarrow 0$

COUNTING MECHANISM

Numerical Methods for solving equations

NEWTON-RAPHSON

$f(x)=0$

TABLES

DIFFERENTIATE

X-INTERCEPT

BISECTION

X_0

ESTIMATES ALWAYS BRACKET THE ROOT

ONE INITIAL ESTIMATE

ITERATION

STOPPING CRITERIA

ROOTS OF A FUNCTION

ESTIMATE

SECANT

ACCURATE TO 2dp

CONVERGENCE

FORMULAE

SOLVING EQUATIONS

$$|x_{n-1} - x_n| < E$$