

Example Context Elaboration: Counters

Focus: Tree diagrams

Achievement objective S7-4

In a range of meaningful contexts, students will be engaged in thinking mathematically and statistically. They will solve problems and model situations that require them to:

Investigate situations that involve elements of chance:

A comparing theoretical continuous distributions, such as the normal distribution, with experimental distributions

B calculating probabilities, using such tools as two-way tables, tree diagrams, simulations, and technology

Counters

Introduction: Ana and Levi are playing a game which involves drawing counters out of a bag.

Problem



If Ana wins when the counters are the same colour and Levi wins when the counters are different colours, is the game fair? How do other similar games work?

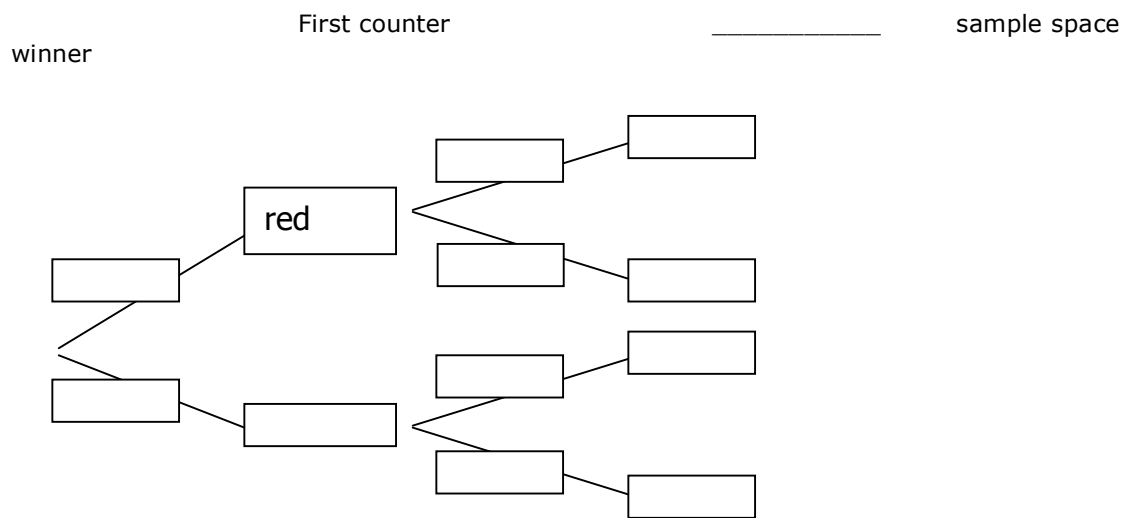
Plan



There are 2 red counters and 3 blue counters. Ana takes one counter out, notes the colour and replaces it. Levi does the same, and they record their results in a table. If both the counters are the same colour, then Ana wins. If the counters are different colours, Levi wins.

They make an outcome tree to represent all outcomes of the situation, and work out the probabilities of two red, two blues, first a red then a blue, first blue then red. The teacher asks them to think about how they could modify a tree to represent the same outcome more simply, and gives them a template:

Plan



Ana and Levi consider what information they could put in the blanks to represent the situation. The teacher asks them to start by thinking just about Ana's choice. Levi remembers that the probabilities of a choice must add to one. They realise that they can use fractions to put the probabilities on the branches. After writing in the fractions and comparing them to the event probabilities worked out previously, they realise that probabilities can be calculated by multiplying along the branches. The class discusses what this means in terms of the context of the game. Ana and Levi understand that the tree diagram does represent the same probability situation as the game, and that multiplying along a branch is the same as finding a proportion of a proportion. They then use the tree diagram to find the probability of Ana winning (two the same) or Levi winning (two different) and see that adding probabilities of outcomes gives the probability of the desired event. Ana and Levi investigate the probability of winning similar games with numbers of counters of different colours, and draw tree diagrams to represent each game.

Analysis



They answer questions such as:

- What number of counters of each colour would make the game fair?
- Is the chance of Levi winning the same with 2 red and 2 blue as it is with 3 red and 3 blue?
- What is the probability of Ana winning if there are 1 red, 1 blue and 1 yellow?
- If there is 1 blue and 1 yellow, how many red would there have to be for Ana to win at least half the time?

Conclusion



Ana and Levi realise that the game is not fair. Ana is more likely to win than Levi.
 $P(\text{Ana win}) = 13/25$, $P(\text{Levi win}) = 12/25$.

Reflection

How does the tree diagram help us to organise the outcomes from a probability event?

Extension activity

How would the game results change if the first counter drawn out was not replaced?