Example Context Elaboration: Armspans

Focus: Normal distribution

Achievement objective S7-4

In a range of meaningful contexts, students will be engaged in thinking mathematically and statistically. They will solve problems and model situations that require them to:

Investigate situations that involve elements of chance:

A comparing theoretical continuous distributions, such as the normal distribution, with experimental distributions

B calculating probabilities, using such tools as two-way tables, tree diagrams, simulations, and technology

Armspans

The class considers the shape of distributions they would expect from a sample of measurements of parts of the body and make some predictions of what they think the distributions will look like.

Problem

What does the distribution of armspans and other body measurements look like and how can this be used to estimate proportions and probabilities back in the population?

Plan

Lee sketches the shape of a predicted distribution of armspans, drawing a symmetrical hatplot.

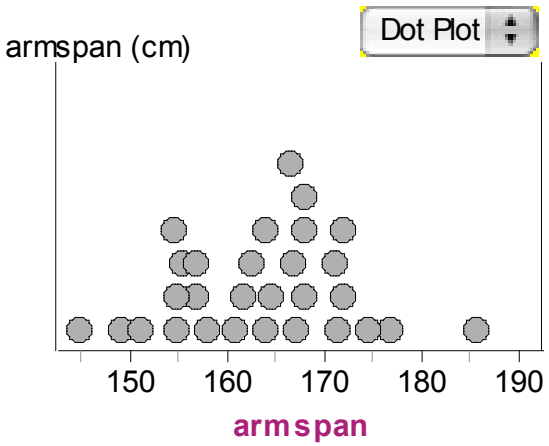
Data

The students help each other to measure their armspans.

Analysis

The class creates a dotplot of armspan measurements on the whiteboard and compares it to their predictions. Lee notices that the class distribution is roughly symmetrical as predicted, but has more values closer to the median than the prediction hatplot (if the class armspan data is not approximately normally distributed, a different set of data will have to be collected).

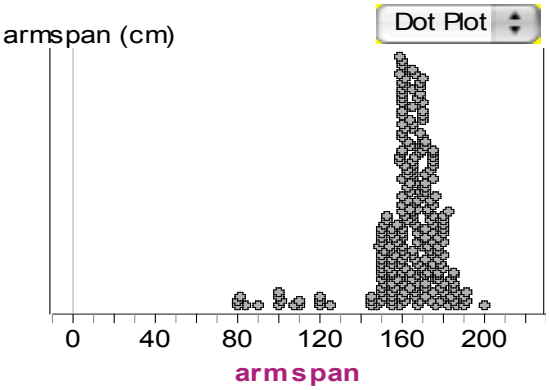
The class calculates the proportion of the class with an armspan over 170 cm They repeat several similar calculations. They find the mean and standard deviation of the class data. They draw this on the graphs and check that it looks ok.



Plan

The class looks at a larger sample of armspans from CensusAtSchool www.censusatschool.org.nz . They decide that the measures less than 130 cm are likely to be errors, so consider only the armspans over 130cm.

Analysis



Lee notices that the distribution is fairly symmetrical, with most values close to the median.

Plan

The class looks at distributions of other large samples of other measurement data from CensusAtSchool: height, right foot length (all databases), neck and wrist circumference (2005 and 2009 databases) and popliteal length (2009 database).

Analysis

Lee notices that these distributions show similar characteristics of symmetry with most values close to the median. As well they consider other features such as clusters, outliers, symmetry and other aspects of shape.

Plan

The teacher asks the students to estimate the length of a line drawn on the board, and graphs the distribution of the estimates

Analysis

Lee notes that the errors in measurements show a similar distribution to the body part measurements.

The teacher introduces the concept of modeling a distribution of data using a theoretical normal distribution, and introduces the proportional properties of the normal distribution.

Lee and Sam are given a set of cards with dot plots of measurement data on them (see master at end of document) to sort out those which might be modeled by a normal distribution.

Going back to the class armspan data, Lee and Sam use the normal distribution to answer the same questions they answered previously, comparing the answers and finding that the proportions are similar.

Lee uses the normal distribution to answer the same questions about the larger CensusAtSchool sample of armspan data, and concludes that the normal distribution gives approximately the same proportions as those found directly from the sample.

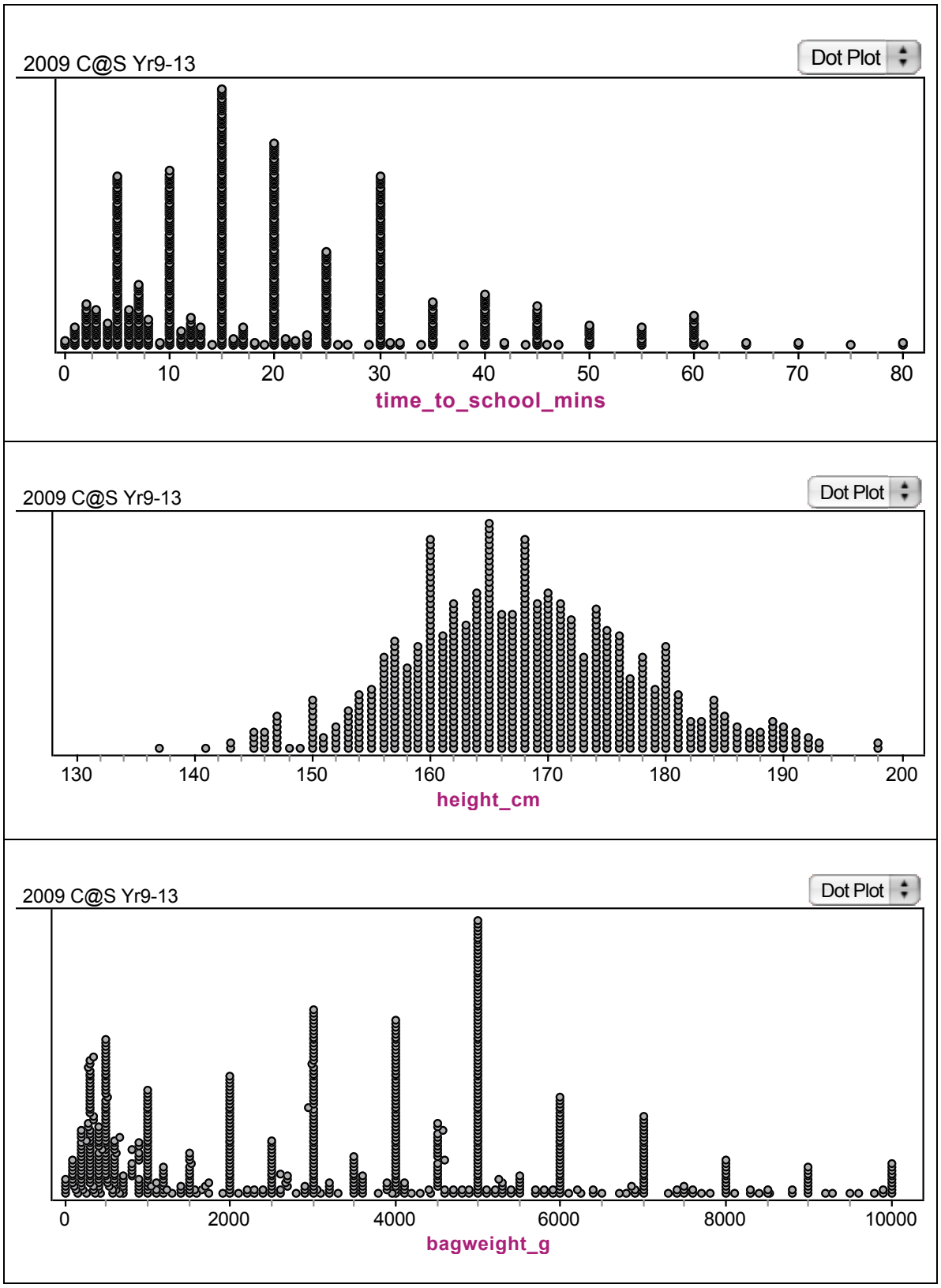
Conclusion

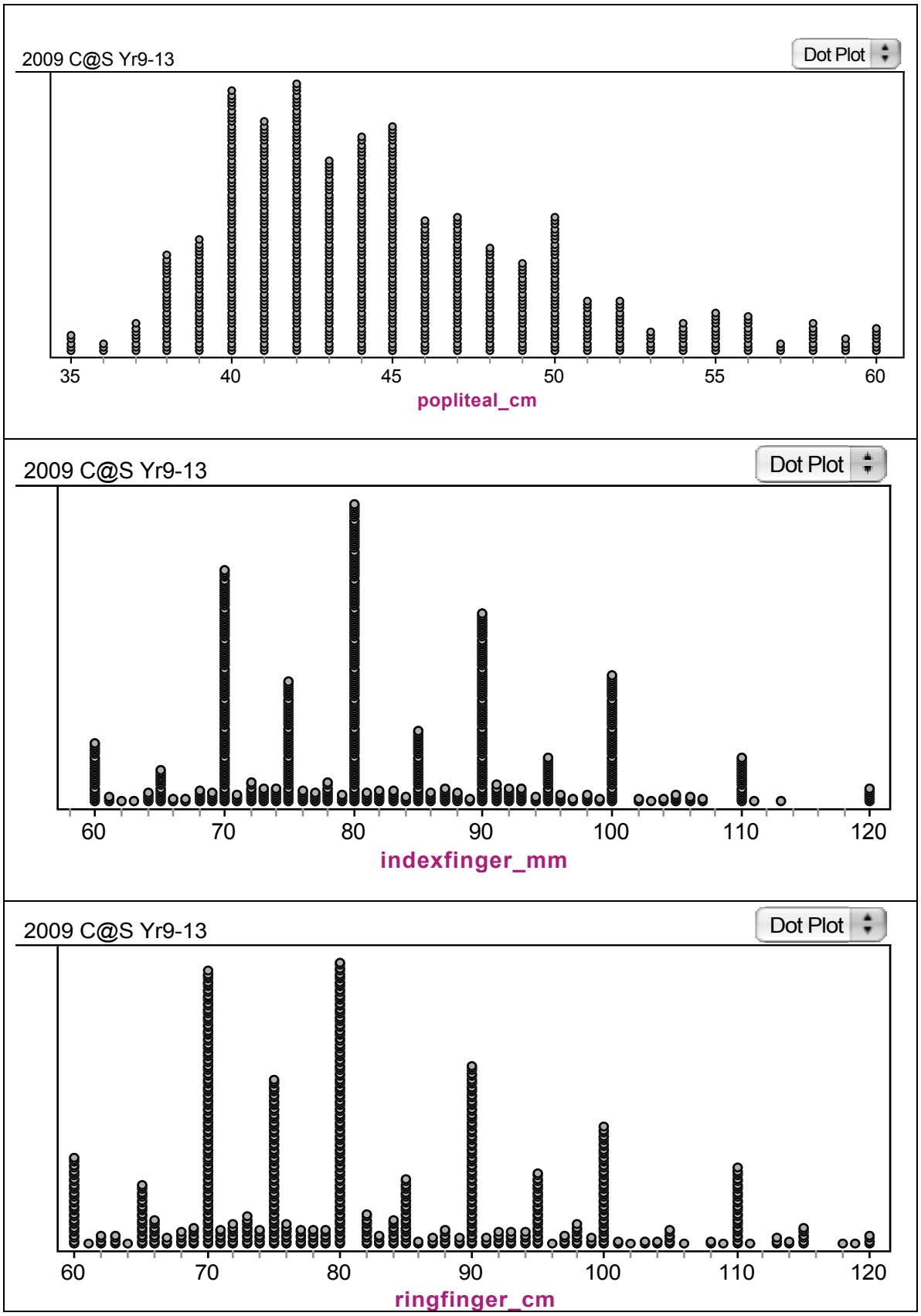
Lee then decides that the normal distribution is a useful model for estimating proportions and probabilities within a population.

Reflection

What does it mean for a distribution to be approximately normal?

Extension activity


2009 C@S Yr9-13
Dot Plot
0
10
20
30
40
50
60
70
80
time_to_school_mins
2009 C@S Yr9-13
Dot Plot
130
140
150
160
170
180
190
200
height_cm
2009 C@S Yr9-13
Dot Plot
0
2000
4000
6000
8000
10000
bagweight_g


2009 C@S Yr9-13
Dot Plot
35
40
45
50
55
60
popliteal_cm
2009 C@S Yr9-13
Dot Plot
60
70
80
90
100
110
120
indexfinger_mm
2009 C@S Yr9-13
Dot Plot
60
70
80
90
100
110
120
ringfinger_cm