Example Context Elaboration: Permutations

Focus: Applying distributions

Achievement objective S8-4

In a range of meaningful contexts, students will be engaged in thinking mathematically and statistically. They will solve problems and model situations that require them to:

Investigate situations that involve elements of chance:

A calculating probabilities of independent, combined, and conditional events

B calculating and interpreting expected values and standard deviations of discrete random variables

C applying distributions such as the Poisson, binomial, and normal

Exploring Music Track Permutations

Arvind and Sheba have three different coloured squares representing three songs on an iPod. They list the number of ways they can listen to the three songs. They extend the idea to five songs and develop the idea of factorials, with n! for the number of ways of ordering n objects using the multiplication principle.

They then consider the possible ways of listening to three of the five tracks, and hence develop a formula for calculating the number of permutations of r items selected from n realizing that the order of playing the tracks is important. They use the multiplication principle for independent events to calculate the number of ways.

$$5 \times 4 \times 3 = 60$$

To confirm the idea they list the number of ways a family which borrows 10 DVDs can watch four of them on one day.

$$10 \times 9 \times 8 \times 7$$

Their teacher helps them to generalize the formula for ${}^{n}P_{r}$ using factorials.

They confirm by enumerating the outcomes for simple cases.

Arvind and Sheba then work on calculating the number of ways a teacher can select three students out of five to go and collect books from the library. They realize that order does not matter in this situation, but that it is related to the number of permutations of three objects selected form 5. They note that there are six ways of ordering the group A,B and C, and six ways of ordering each of the groups of three, i.e. 3!.

Selections of 3 items from ABCDE
ABC
ACB
BAC
BCA
CBA
CAB
6 permutations of three items

They compare this with the number of permutations and develop the formula for ^nC_r leading to the formula for Combinations. These values can be related to the binomial coefficients in Pascal's triangle, and confirmed using technology.