Example Context Elaboration: Binomial Counters

Focus: Applying distributions

Achievement objective S8-4

In a range of meaningful contexts, students will be engaged in thinking mathematically and statistically. They will solve problems and model situations that require them to:

Investigate situations that involve elements of chance:

A calculating probabilities of independent, combined, and conditional events

B calculating and interpreting expected values and standard deviations of discrete random variables

C applying distributions such as the Poisson, binomial, and normal

Counter Game Simulation

David and Jonah each select five counters from a bag containing a large number of counters in two colours, say red and blue, where the proportion of red counters is known, replacing the counters after each withdrawal. They each count up the number of red counters in their selection of ten, and record the result. The player who has the higher number of red counters wins a point. If both players have the same number of red counters, neither scores a point. David and Jonah keep playing until one of them reaches 20 points. (This could be regarded as a simulation, for example, two traffic police checking for warrant of fitness, where a red counter represents no warrant of fitness, blue represents a current warrant.)

Recording sheet

# of red counters	0	1	2	3	4	5
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experimental probability						

David and Jonah use their results to calculate the experimental probability for each of the possible outcomes and draw a column graph of their results. They observe the shape and note that it appears to be approximately symmetrical about some average value.

After conducting the experiment David and Jonah estimate the proportion of red counters in the bag, and discuss how many times they would need to repeat the experiment in order to be confident that their estimate was correct.

David and Jonah then calculate the **theoretical probability** for each outcome, using a probability tree and their estimate of the population proportion of red counters (or the actual proportion given by their teacher) or p. Alternatively, they could use the principles developed during explorations of combinations. They can check their values using their calculator.

# of red counters	0	1	2	3	4	5
theoretical probability	$(1-p)^5$	$5p(1-p)^4$	$10p^2(1-p)^3$	$10p^3(1-p)^2$	$5p^4(1-p)$	p^5

David and Jonah discuss the patterns they can see in the theoretical probabilities, leading to the general formula for k successes out of n trials

$$\text{Prob } (X = k) = {}^nC_k\, p^k\, (1-p)^k$$

Their teacher leads them to the realization that the coefficients 1, 5, 10, 10, 5, 1 are the binomial coefficients from Pascal's triangle. David and Jonah compare the column graph for their experimental probabilities with a column graph of the theoretical probabilities.

Discussing with the teacher, they could tease out the conditions for a binomial distribution to be a suitable model, that is:

- Two possible outcomes, success or failure
- A fixed number of trials, n
- A fixed probability of success, p
- Each trial independent of the others