Example Context Elaboration: Unusual Dice

Focus: Standard deviation

Achievement objective S8-4

In a range of meaningful contexts, students will be engaged in thinking mathematically and statistically. They will solve problems and model situations that require them to:

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Investigate situations that involve elements of chance:

A calculating probabilities of independent, combined, and conditional events

B calculating and interpreting expected values and standard deviations of discrete random variables

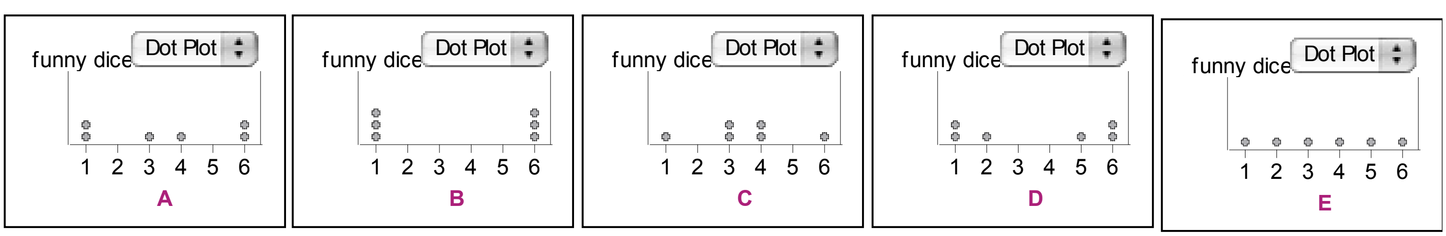
C applying distributions such as the Poisson, binomial, and normal

Dice Distribution

Their teacher asks Laura and Bethany to consider the probability distribution of some unusual six-sided dice, labeled A to E.

A has two faces labeled 1, one face labeled 3, one face labeled 4 and two faces labeled 6.
B has three faces labeled 1 and three faces labeled 6.
C has one face labeled 1, two faces labeled 3, two faces labeled 4 and one face labeled 6.
D has two faces labeled 1, one face labeled 2, one face labeled 5 and two faces labeled 6.
E is a normal six-sided die.

Laura and Bethany draw dot plots to show the probabilities of the possible outcomes on each die. They notice that all five distributions have the same mean and range, but the shape of the distributions differs.



They notice that the spread of the possible outcomes varies even though the range is the same. The teacher discusses with them the possibility of having a measure of spread for the distribution which takes into account this spread of possible outcomes, based on the distance of each outcome from the mean, in the same way as a statistical distribution. Using technology, Laura and Bethany calculate the mean and standard deviation for each probability distribution. They mark the mean and standard deviation on their graphs. They make comments about what they notice (for example, from looking at the graphs of the distributions could they predict the location of the mean, and rank the distributions on standard deviation from smallest to largest?).

	A	B	C	D	E
mean, E(*X*)	3.5	3.5	3.5	3.5	3.5
Range (X)	1 to 6	1 to 6	1 to 6	1 to 6	1 to 6
SD(*X*)	2.0615	2.5	1.5	2.2174	1.7078

Laura and Bethany note that the higher the probability is for the extreme values of the random variable, the greater the value of the standard deviation. The teacher may take them through the theoretical calculation, showing that the standard deviation is the square root of the expected squared distance of X from its mean which can be regarded as a rough measure of the average distance from the mean.

Expected squared distance of X from the mean, μ = $\sum (x-\mu)^2 P(X=x)$

Standard deviation of X, SD(X) = $\sqrt{\sum (x-\mu)^2 P(X=x)}$ for all possible values of *x.*

They then explore the mean and standard deviation for other situations, observing that the standard deviation averages out the distance from the mean of each possible value of the random variable.

Their teacher leads them into real data that looks similar, for example, X = number of siblings each has; X= number of quoits you can get on a spike (given 6 quoits), X= number of kid's blocks you can stack onto a tower before it falls. Find μ, SD(X) for all.

Teachers should note that the emphasis has shifted away from variance to standard deviation. The level of applicability in the real world of linear combinations and transformations of random variables does not warrant their inclusion in the curriculum.