Example Context Elaboration: Game of Hog

Focus: Expected value

Achievement objective S8-4

In a range of meaningful contexts, students will be engaged in thinking mathematically and statistically. They will solve problems and model situations that require them to:

Investigate situations that involve elements of chance:

A calculating probabilities of independent, combined, and conditional events

B calculating and interpreting expected values and standard deviations of discrete random variables

C applying distributions such as the Poisson, binomial, and normal

Expected Value of a Dice Game

Laura and Bethany play the game of Hog. They throw two dice, one red and one blue, taking it in turns. If a one shows, they score zero, otherwise they score the total showing on the dice. They decide they will play until one of them reaches one hundred points. They wonder how many games they will have to play. They realize that the number of games they will need to play depends on the average score per turn.

They play the game until one of them wins, keeping a tally of the scores they throw. This gives them a sample of one to estimate the number of games on average they will have to play for one of them to win. They realize that it will take a long time to do enough experiments to make a reliable estimate. They decide to use probability to help them work out their average score for each turn. They construct a table of possible scores:

		score on red die					
		1	2	3	4	5	6
score on blue die	1	0	0	0	0	0	0
	2	0	4	5	6	7	8
	3	0	5	6	7	8	9
	4	0	6	7	8	9	10
	5	0	7	8	9	10	11
	6	0	8	9	10	11	12

Their teacher shows them how to construct the probability function for their scores, with all possible scores listed in the top row, with their respective probabilities underneath in the second row. Laura and Bethany realize that they can use the method developed above to calculate the mean (expected value) of the score for each turn.

possible score, x	0	4	5	6	7	8	9	10	11	12
probability	11/36	1/36	2/36	3/36	4/36	5/36	4/36	3/36	2/36	1/36
probability	0.2821	0.0256	0.0513	0.0769	0.1026	0.1282	0.1026	0.0769	0.0513	0.0256

Students draw a graph for this probability distribution, comment on what they notice, and visually predict the location of the expected value of a turn.

Their calculation produces a value of 5.556 (3dp) for the expected value of a turn. From this Laura and Bethany are able to calculate that on average they will need about 100 / 5.556 = 18 turns to get 100 points.